

Patching up the No-Boundary Proposal with virtual Euclidean wormholes

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Abstract

In quantum cosmology, one often considers tunneling phenomena which may have occurred in the early universe. Processes requiring quantum penetration of a potential barrier include black hole pair creation and the decay of vacuum domain walls. Ideally, one calculates the rates for such processes by finding an instanton, or Euclidean solution of the field equations, which interpolates between the initial and final states. In practice, however, it has become customary to calculate such amplitudes using the No-Boundary Proposal of Hartle and Hawking. A criticism of this method is that it does not use a single path which interpolates between the initial and final states, but two disjoint instantons: One divides the probability to create the final state from nothing by the probability to create the initial state from nothing and decrees the answer to be the rate of tunneling from the initial to the final state. Here, we demonstrate the validity of this approach by constructing continuous paths connecting the ingoing and outgoing data, which may be viewed as perturbations of the set of disconnected instantons. They are off-shell, but will still dominate the path integral as they have action arbitrarily close to the no-boundary action. In this picture, a virtual domain wall, or wormhole, is created and annihilated in such a way as to interface between the disjoint instantons. Decay rates calculated using our construction differ from decay rates calculated using the No-Boundary Proposal only in the prefactor; the exponent, which usually dominates the result, remains unchanged.

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I. INTRODUCTION

Ripple in still water
When there is no pebble tossed
Nor wind to blow
-*The Grateful Dead*

Originally [1], the No-Boundary Proposal was an attempt to eliminate the initial and final singularities (which are ‘boundaries’ for spacetime in cosmological scenarios), by considering the universe as a history in imaginary time. One is then led to a picture of a universe which is finite in imaginary time and ‘without boundary’. This picture involves both ‘real time’ (or Lorentzian) and ‘imaginary time’ (or Euclidean) sections of the full complexified spacetime. When one combines this picture with the sum-over-histories prescription for calculating amplitudes, it is natural to think of a Euclidean section (which matches smoothly to a Lorentzian section across a spacelike three-surface of vanishing extrinsic curvature) as an ‘instanton’, which mediates the creation of the Lorentzian section from ‘nothing’. In this way, a new universe can appear from ‘nothingness’, even though there was no source around to precipitate such an event. One then speaks of creating universes from nothing. Similarly, one can consider the time reverse and speak of universes ‘annihilating’ to nothingness. One can calculate Euclidean actions of the relevant instantons and obtain the rate at which various universes appear from nothing. One can thus ‘predict’ the most likely initial state for the universe. The key point about this version of the No-Boundary Proposal is that it tells you how to calculate the probability that *something* appears from *nothing*.

Here, we are concerned with another variant of the No-Boundary Proposal, which tells you how to calculate the rate at which *something* decays into *something else*. This version of the proposal is often applied to the study of black hole pair creation. Throughout this paper we use units in which $\hbar = c = G = 1$.

II. APPROACHES TO GRAVITATIONAL TUNNELING

The pair creation of black holes, first discovered by Gibbons [2], has been studied enthusiastically for a number of years. It corresponds to a non-perturbative, topological fluctuation of the gravitational field. As such it is one of the few effects of quantum gravity that one can hope to study quantitatively in a semi-classical approximation. It has been used to investigate the entropy of black holes [3–5], and electric-magnetic duality in quantum gravity [6]. In the cosmological context, it has clarified the important role of the Hartle-Hawking No-Boundary Proposal in quantum gravity [7, 8], and it may have profound consequences for the global structure of the universe [9].

A. Bounce Approach

Black hole pair creation can be analyzed semi-classically by the use of instanton methods. Typically, the nucleation process is described by a single Euclidean solution of the Einstein equations, a bounce. It interpolates between an initial spacelike section without black holes,

and a final spacelike section containing black holes, bouncing back to the initial section in a time-symmetric fashion. An instanton is half a bounce, i.e. a geometry connecting initial and final spacelike sections, but not bouncing back. One calculates the bounce action, I_{pc} , which must be renormalized by subtracting off the action I_{bg} of a Euclidean geometry containing only background spacelike sections. Thus one obtains the pair creation rate Γ :

$$\Gamma = \exp[-(I_{\text{pc}} - I_{\text{bg}})], \quad (2.1)$$

where we neglect a prefactor. Note that both I_{pc} and I_{bg} are typically infinite, but their difference can be made well defined and finite in a suitable limit. This prescription has been used very successfully by a number of authors [3–6, 10–12] for the pair creation of black holes on various backgrounds. It is motivated by analogies in quantum mechanics and quantum field theory [13, 14], as well as by considerations of black hole entropy [3–5].

B. Quantum Cosmological Approach

A positive cosmological constant, or a domain wall, typically causes the universe to close. In these situations, there are Lorentzian solutions with and without black holes, but there are no known Euclidean solutions connecting their spacelike slices. Instead, there are two separate, compact Euclidean geometries corresponding to creation from nothing of a universe with, and without a black hole pair. Thus the instanton technique outlined above could not be used directly. Before describing how virtual domain walls mend this problem, we review how the problem was circumvented using concepts from quantum cosmology.

In quantum cosmology one works with the concept of the *wave function of the universe*. The wave function takes different values for a universe with, and without black holes. The squared amplitude of the wave function yields a probability measure. According to the Hartle-Hawking No-Boundary Proposal, the wave function of the universe, evaluated for a specified three-geometry, is given by a path integral over all closed, smooth complex geometries that match the specified boundary conditions on the spacelike section, and have no other boundary; the integrand is the exponential of minus the Euclidean action of the geometry. In the semi-classical approximation, the wave function is approximated as

$$\Psi = e^{-I_{\text{inst}}}, \quad (2.2)$$

where I_{inst} is the action of a saddlepoint solution which satisfies the Einstein equations under the given boundary conditions. If there is no such instanton, the wave function will be zero semi-classically; if there are several, they have to be summed over. The probability measure for a given universe is thus related to the action of an instanton which describes the nucleation of the universe from nothing:

$$P = \Psi^* \Psi = e^{-2I_{\text{inst}}^{\text{Re}}}. \quad (2.3)$$

Clearly, the probability depends only on the real part of the instanton action, $I_{\text{inst}}^{\text{Re}}$. We shall see in the next section that there are, indeed, two instantons, each of which nucleates a universe from nothing. One will lead to spacelike sections with black holes, the other to an empty background universe. Because of the cosmological term, the Euclidean geometry

is compact, and the actions of both instantons are finite. Thus probability measures can be assigned to a universe with, and without black holes.

But how are these probability measures related to pair creation? It seems that all one can do in quantum cosmology is to compare the probability measure for a universe with *one* pair of black holes to that of an empty universe. The black hole instanton would then be without any cosmological relevance whatsoever – it could only produce a single black hole pair in an exponentially large universe. It is possible, however, to propose the following approach [8]: Consider an arbitrary Hubble volume in an inflating universe. Typically, this volume will not contain black holes; it will be similar to a Hubble volume of de Sitter space. After one Hubble time, its spatial volume will have increased by a factor of $e^3 \approx 20$. By the de Sitter no hair theorem, one can regard each of these 20 Hubble volumes as having been nucleated independently [15], through either the empty, or the black hole instanton. Thus, one allows for black hole pair creation, since some of the new Hubble volumes may contain black holes. We shall see later that the spacelike sections can be taken to be three-spheres in the case of a universe without black holes, and $S^1 \times S^2$ for a universe with a single pair of black holes. We will therefore compare the wave functions for spacelike slices with these two topologies. Using the No-Boundary Proposal [1], one can assign probability measures to both instanton types. The ratio of the probability measures,

$$\Gamma = \frac{P_{\text{BH}}}{P_{\text{no BH}}}, \quad (2.4)$$

reflects the ratio of the number of Hubble volumes containing black holes, to the number of empty Hubble volumes. This is true as long as this ratio is very small, so that the holes will be widely separated. But we shall see that this condition is satisfied whenever the semi-classical approximation is valid. Since this argument applies to every new generation of Hubble volumes, the ratio Γ is the number of black hole pairs produced per Hubble volume, per Hubble time. In other words, Γ is the rate of black hole pair creation in inflation.

C. The Cosmological Pair Creation Instantons

We now illustrate this approach by reviewing its implementation for the case of neutral black holes created on a cosmological background [7, 8, 16]. We begin with the simpler of the two spacetimes, an inflationary universe without black holes, where the spacelike sections are round three-spheres. In the Euclidean de Sitter solution, the three-spheres begin at zero radius, expand and then contract in Euclidean time. Thus they form a four-sphere of radius $\sqrt{3/\Lambda}$. The analytic continuation can be visualized (see Fig. 1) as cutting the four-sphere in half, and then joining to it half the Lorentzian de Sitter hyperboloid. The real part of the Euclidean action for this geometry comes from the half-four-sphere only: $I_{\text{no BH}}^{\text{Re}} = -3\pi/2\Lambda$. Thus, the probability measure for de Sitter space is

$$P_{\text{no BH}} = \exp\left(\frac{3\pi}{\Lambda}\right). \quad (2.5)$$

FIGURES

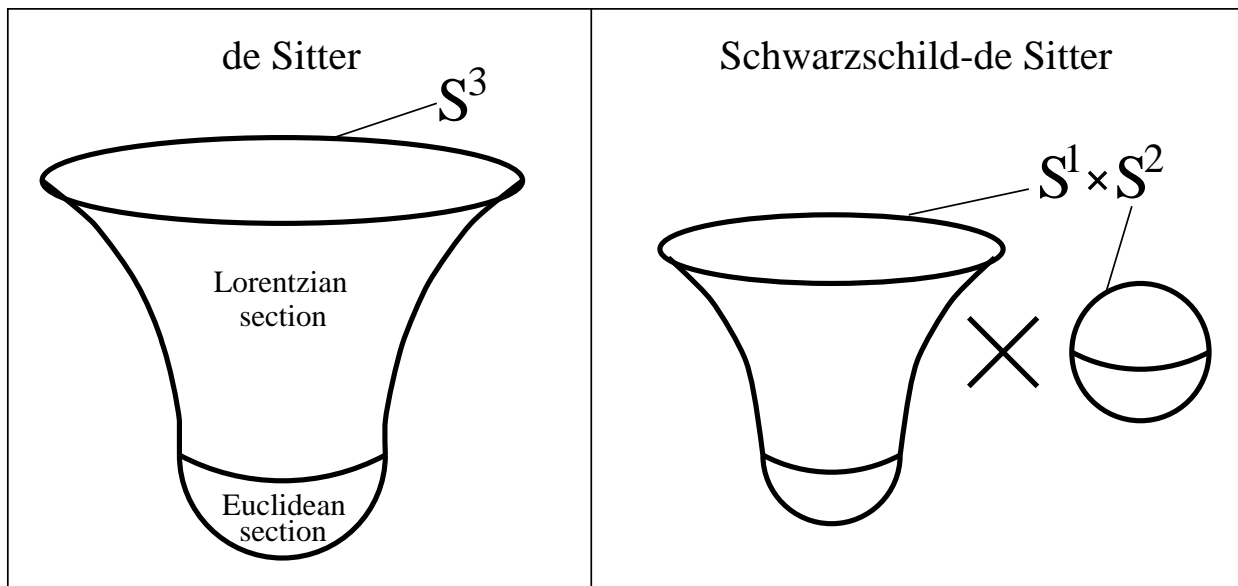


FIG. 1. The creation of a de Sitter universe (left) can be visualized as half of a Euclidean four-sphere joined to a Lorentzian four-hyperboloid. The picture on the right shows the corresponding nucleation process for a de Sitter universe containing a pair of black holes. In this case the spacelike slices have non-trivial topology.

Now we need to go through the same procedure with the Schwarzschild-de Sitter solution, which corresponds to a pair of black holes immersed in de Sitter space. Its Lorentzian metric is given by

$$ds^2 = -V(r)dt^2 + V(r)^{-1}dr^2 + r^2d\Omega^2, \quad (2.6)$$

where

$$V(r) = 1 - \frac{2\mu}{r} - \frac{\Lambda}{3}r^2. \quad (2.7)$$

Here μ parameterizes the mass of the black hole, and for $\mu = 0$ the metric reduces to de Sitter space. The spacelike sections have the topology $S^1 \times S^2$. This can be seen by the following analogy: Empty Minkowski space has spacelike sections of topology \mathbf{R}^3 . Inserting a black hole changes the topology to $S^2 \times \mathbf{R}$. Similarly, if we start with de Sitter space (topology S^3), inserting a black hole is like punching a hole through the three-sphere, thus changing the topology to $S^1 \times S^2$. In general, the radius of the S^2 varies along the S^1 . In the static slicing of Schwarzschild-de Sitter, the maximum two-sphere corresponds to the cosmological horizon, the minimum to the black hole horizon. This is shown in Fig. 2.

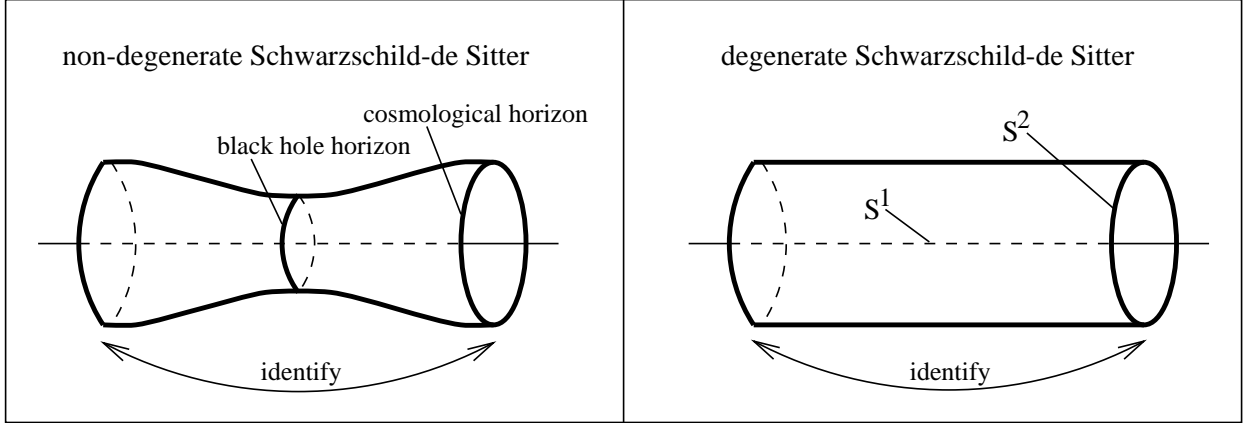


FIG. 2. The spacelike slices of Schwarzschild-de Sitter space have the topology $S^1 \times S^2$. In general (left), the size of the two-sphere varies along the one-sphere. If the black hole mass is maximal, however, all the two-spheres have the same size (right). Only in this case is a smooth Euclidean solution admitted.

What we need is a Euclidean solution that can be analytically continued to contain this kind of spacelike slice. It turns out that such a smooth instanton does not exist in general for the Lorentzian Schwarzschild-de Sitter spacetimes. The only exception is the degenerate case, where the black hole has the maximum possible size, and the radius of the two-spheres is constant along the S^1 (see Fig. 2). The corresponding Euclidean solution is just the topological product of two round two-spheres, both of radius $1/\sqrt{\Lambda}$ [16]. It can be analytically continued to the Lorentzian Schwarzschild-de Sitter solution by cutting one of the two-spheres in half, and joining to it the 2-dimensional hyperboloid of $1+1$ dimensional Lorentzian de Sitter space, as shown in Fig. 1. In the Lorentzian regime the S^1 expands exponentially, while the two-sphere just retains its constant radius. Thus, unless more sophisticated boundary conditions are employed [17], the Euclidean approach predicts the nucleation of black holes of the maximum size, $r_{\text{BH}} = \Lambda^{-1/2}$.

The real part of the Euclidean action for this instanton is given by $I_{\text{BH}}^{\text{Re}} = -\pi/\Lambda$, and the corresponding probability measure is

$$P_{\text{BH}} = \exp\left(\frac{2\pi}{\Lambda}\right). \quad (2.8)$$

Now we can take the ratio of the two probability measures, and obtain the pair creation rate:

$$\Gamma = \exp\left(-\frac{\pi}{\Lambda}\right). \quad (2.9)$$

This example illustrates the analogy between the standard prescription for pair creation, Eq. 2.1, and the result obtained from the No-Boundary Proposal: By Eqs. (2.4) and (2.3),

$$\Gamma = \frac{P_{\text{BH}}}{P_{\text{no BH}}} = \exp\left[-\left(2I_{\text{BH}}^{\text{Re}} - 2I_{\text{no BH}}^{\text{Re}}\right)\right], \quad (2.10)$$

where I^{Re} denotes the real part of the Euclidean action of a nucleation geometry. But we have seen that the only contribution to I^{Re} comes from the action of the Euclidean sector

of the nucleation geometry, the instanton. This, in turn, is equal to half of the action of the complete bounce solution, which is used in the usual pair creation framework. Thus $I_{\text{pc}} = 2I_{S^1 \times S^2}^{\text{Re}}$ and $I_{\text{bg}} = 2I_{S^3}^{\text{Re}}$, and we recover Eq. 2.1.

The quantum cosmological approach to black hole pair creation outlined above clearly differs from the usual bounce approach. In the latter, a Euclidean time region smoothly interpolates between the two different spacelike slices; in quantum cosmology, on the other hand, one can think of the pair creation process as the annihilation of a Hubble volume, and its subsequent recreation with a different spatial topology. Thus it is, perhaps, less obvious to see the analogy to quantum mechanical tunneling instantons that motivate the bounce approach to pair creation. Some have therefore regarded cosmological and domain wall pair creation scenarios with a high degree of scepticism. While we have always found the probabilistic argument given above convincing, and thought it justified to calculate a pair creation rate from the No-Boundary Proposal, we hope to allay any further worries by explicitly constructing an interpolating tunneling path using virtual wormholes, or domain walls. It will be shown that the wormhole contribution to the Euclidean action can be negligible. The quantum cosmological formula for the pair creation rate, Eq. (2.4), will thus be confirmed.

D. The Patching Proposal

As we have discussed above, when one uses the No-Boundary Proposal to calculate a tunneling amplitude one does not actually construct an imaginary time resonance connecting the initial and final states. In fact, in many scenarios involving topology change, there simply *does not exist* any globally regular solution of the relevant Euclidean equations of motion which interpolates between ingoing and outgoing data. There are thus two problems: First, if one takes the point of view that disconnected geometries should not be allowed in the path integral, one would have to exclude the solutions given in the previous section. Then it seems that there would be no saddle point, and the transition rate should vanish semi-classically. Second, even if disconnected geometries are admitted, it is not immediately obvious why the actions of the two disjoint instantons should be subtracted, rather than added. Here we make a proposal that solves both problems: the idea is to join the disconnected geometries by small virtual domain walls.

In the path integral approach, one obtains an amplitude by summing over *all* paths, whether they are solutions or not. Given a path integral with one saddlepoint, it will be of interest to consider the consequences of taking the integral over all paths *except* for the saddlepoint and very small perturbations about it. If the removed region is sufficiently small, there will still be a region of stationary action, located around the excised area. There the oscillations of the integrand will not be destructive and the amplitudes will add up. Paths which are close to being solutions will then dominate the sum.

To make this precise, let us assume that the saddlepoint action is given by I_0 , and consider perturbations of this solution parameterized by δ . Then the action near the saddlepoint will be given by

$$I(\delta) = I_0 + \frac{1}{2}\rho\delta^2, \quad (2.11)$$

where ρ is the second derivative of the action evaluated at the saddlepoint. Ignoring other perturbations, the path integral will be given by

$$\int_{-\infty}^{\infty} d\delta e^{-I} = e^{-I_0} \int_{-\infty}^{\infty} d\delta e^{-\frac{1}{2}\rho\delta^2} \quad (2.12)$$

$$= \sqrt{\frac{2\pi}{\rho}} e^{-I_0} \quad (2.13)$$

in the saddlepoint approximation.

Our idea is the following: We take the saddlepoint geometry to be the combination of a half-four-sphere (which annihilates a de Sitter Hubble volume) with half of an $S^2 \times S^2$, which will create Schwarzschild-de Sitter space from nothing. One may connect these two disjoint instantons by removing a small four-ball of radius δ on each and joining them together on the resulting boundaries. This leads to a family of near-solutions, in which the instantons are connected though a virtual domain wall of size δ . These geometries, which violate Einstein's equations (with positive energy sources) in a small region, will actually interpolate between the initial and final spacelike sections. The disjoint saddlepoint solution is recovered in the limit where $\delta \rightarrow 0$. The idea works just the same for the disjoint instantons associated with black hole pair creation on (real) domain walls, which will be discussed below.

One must assume that the virtual domain walls may not be smaller than Planck size: $|\delta| > 1$. Forbidding the disconnected geometries thus corresponds to removing the region $|\delta| < 1$ from the range of integration in Eq. (2.12). By calculating the action contribution of the virtual domain wall, we will show that ρ is of order one. Thus we are excising a region of about one standard deviation from the integral. This will reduce the prefactor of the wave function to about a third of its value in Eq. (2.13). The exponent, which typically is much more significant, will not change at all. (Since δ should be small compared to the size of the instanton, there will strictly also be an upper bound. But the overwhelming contribution to the integral comes from the first few standard deviations. Therefore the error from integrating to infinity will be negligible except for Planck scale instantons, when the semiclassical approach breaks down anyway).

Therefore, connected geometries will dominate the path integral in the absence of disconnected ones. This solves the first problem raised at the beginning of this subsection. The second problem is resolved by the change of orientation at the virtual domain wall, which will cause the two instanton actions to enter the exponent with opposite sign.

The construction of the off-shell interpolating Euclidean paths will be presented in detail in Sec. IV. In order to be precise, we will explicitly go through our construction for the scenario where black holes are pair produced in the background of a Vilenkin-IPser-Sikivie domain wall, as discussed in [18]. Our motivation for treating the tunneling process of black hole pair production in the presence of a domain wall is twofold. First of all, we are going to have to introduce the notion of a domain wall, or infinitely thin wormhole, anyway in order to implement our proposal. Second of all, the qualitative features of black hole pair production by a domain wall are identical to those of black hole pair production in a de Sitter background; indeed, it will not be hard to see that everything we will say here for the domain wall situation will go through for the de Sitter situation, where one is interested in the creation of primordial black holes in the early universe [7, 8, 19]. With all of this in mind, we now present a remedial overview of domain walls.

III. BLACK HOLE PAIR CREATION ON DOMAIN WALLS

A. Domain Walls: A Brief Introduction

A vacuum domain wall is a two-dimensional topological defect which can form whenever there is a breaking of a discrete symmetry. Commonly, one thinks of the symmetry breaking in terms of some Higgs field Φ . If \mathcal{M}_0 denotes the ‘vacuum manifold’ of Φ (i.e., the submanifold of the Higgs field configuration space on which the Higgs acquires a vacuum expectation value because it will minimize the potential energy $V(\Phi)$), then a necessary condition for a domain wall to exist is that $\pi_0(\mathcal{M}_0) \neq 0$. In other words, vacuum domain walls arise whenever the vacuum manifold is not connected. The simplest example of a potential energy which gives rise to vacuum domain walls is the classic ‘double well’ potential, which is discussed in detail (along with many related things) in [20].

(Note: In general, domain walls can arise as (D-2)-dimensional defects (or extended objects) in D-dimensional spacetimes. In fact, domain walls are a common feature in the menagerie of objects which can arise in the low-energy limit of string theory, as has been discussed in detail in [21] and [22].)

From what we have said so far, the Lagrangian density for the matter field Φ is given as [20]

$$\mathcal{L}_m = -\frac{1}{2}g^{\alpha\beta}\partial_\alpha\Phi\partial_\beta\Phi - V(\Phi). \quad (3.1)$$

The exact form of $V(\Phi)$ is not terribly important. All that we require in order for domain walls to be present is that $V(\Phi)$ has a discrete set of degenerate minima, where the potential vanishes. Given this matter content, the full (Lorentzian) Einstein-matter action then reads:

$$S = \int_M d^4x \sqrt{-g} \left[\frac{R}{16\pi} + \mathcal{L}_m \right] + \frac{1}{8\pi} \int_{\partial M} d^3x \sqrt{h} K. \quad (3.2)$$

Here, M denotes the four-volume of the system, and ∂M denotes the boundary of this region. One obtains the Euclidean action, I , for the Euclidean section of this configuration by analytically continuing the metric and fields and reversing the overall sign. The ‘simplified’ form of this Euclidean action in the thin wall limit has been derived in a number of recent papers ([18], [23]) and so we will not reproduce the full argument here. Basically, one first *assumes* that there is no cosmological constant ($R = 0$) and then one uses the fact that the fields appearing in the matter field Lagrangian depend only on the coordinate ‘ z ’ normal to the wall, and one integrates out this z -dependence to obtain the expression

$$I = - \sum_{i=1}^n \frac{\sigma_i}{2} \int_{D_i} d^3x \sqrt{h_i}. \quad (3.3)$$

Here, D_i denotes the i -th domain wall, σ_i is the energy density of the domain wall D_i , h_i is the determinant of the three-dimensional metric $h^{ab}_{(i)}$ induced on the domain wall D_i and n is the total number of domain walls. Now, it is well known that variation relative to $h^{ab}_{(i)}$ on each domain wall will yield the Israel matching conditions. Since we will make use of these conditions, we reproduce them here for the convenience of the reader:

1. A domain wall hypersurface is totally umbilic, i.e., the second fundamental form K_{ij} is proportional to the induced metric h_{ij} on each domain wall world sheet.
2. The discontinuity in the second fundamental form on each domain wall hypersurface is $[K_{ij}]_{\pm} = 4\pi\sigma h_{ij}$.

Thus, the energy density of a thin domain wall measures the jump in the extrinsic curvature of surfaces parallel to the wall as one moves through the wall. We will use these conditions to do quick ‘cut-and-paste’ constructions of virtual domain wall surfaces.

Now, the above the discussion is a nice summary of the field theoretical aspects of a generic vacuum domain wall, but what would a gravitating domain wall actually look like?

B. The VIS Domain Wall Spacetime

Solutions for the gravitational field of a domain wall were found by Vilenkin [24] (for an open wall) and Ipser and Sikivie [25] (for closed walls). The global structure of these Vilenkin-Ipser-Sikivie (or ‘VIS’) domain walls has been extensively discussed recently ([18], [23]) so we will only present a brief sketch here.

To start with, we are looking for a solution of the Einstein equations where the source term is an energy momentum tensor describing a distributional source located at $z = 0$:

$$T_{\mu\nu} = \sigma \delta(x) \text{diag}(1, 1, 1, 0). \quad (3.4)$$

It is not possible to find a static solution of the Einstein equations with this source term; indeed, the VIS solution is a time-dependent solution describing a uniformly accelerating domain wall. In order to understand the global causal structure of the VIS domain wall, it is most useful to use coordinates (t, x, y, z) so that the metric takes the form

$$ds^2 = (1 - k|z|)^2 dt^2 - dz^2 - (1 - k|z|)^2 e^{2kt} (dy^2 + dx^2). \quad (3.5)$$

Here, $k = 2\pi\sigma$. The gravitational field of this solution has unexpected properties. For example, in the Newtonian limit of the Einstein equations for (3.5) one obtains the equation

$$\nabla^2 \phi = -2\pi\sigma,$$

where ϕ is the Newtonian gravitational potential and σ is the energy density of the wall. From this equation it is clear that a wall with *positive* surface energy density will have a repulsive gravitational field, whereas a wall with negative energy density will have an attractive gravitational field. An even simpler way to see that the (positive σ) VIS wall is repulsive is to notice that the $t - z$ part of the metric is just the Rindler metric.

Further information is recovered by noticing that the $z=\text{constant}$ hypersurfaces are all *isometric* to 2 + 1 dimensional de Sitter space:

$$ds^2 = dt^2 - e^{2kt} (dy^2 + dx^2). \quad (3.6)$$

Given that 2 + 1 de Sitter has the topology $S^2 \times \mathbf{R}$ it follows that the domain wall world sheet has this topology. In other words, at each instant of time the domain wall is

topologically a two-dimensional sphere. Indeed, in the original Ipser-Sikivie paper a coordinate transformation was found which takes the (t, x, y, z) coordinates to new coordinates (T, X, Y, Z) such that in the new coordinates the metric becomes (on each side of the domain wall):

$$ds^2 = dT^2 - dX^2 - dY^2 - dZ^2. \quad (3.7)$$

Furthermore the domain wall, which in the old coordinates is a plane located at $z = 0$, is in the new coordinates the hyperboloid

$$X^2 + Y^2 + Z^2 = \frac{1}{k^2} + T^2. \quad (3.8)$$

Of course, the metric induced on a hyperboloid embedded in Minkowski spacetime is just the de Sitter metric, and so this is consistent with what we have already noted. This metric provides us with a useful way of constructing the maximal extension of the domain wall spacetime:

First, take two copies of Minkowski space, and in each copy consider the interior of the hyperboloid determined by equation (3.8). Then match these solid hyperboloids to each other across their respective boundaries; there will be a ridge of curvature (much like the edge of of a lens) along the matching surface, where the domain wall is located. Thus, an inertial observer on one side of the wall will see the domain wall as a sphere which accelerates towards the observer for $T < 0$, stops at $T = 0$ at a radius k^{-1} , then accelerates away for $T > 0$. We illustrate this construction in Fig. 3, where we include the acceleration horizons to emphasize the causal structure.

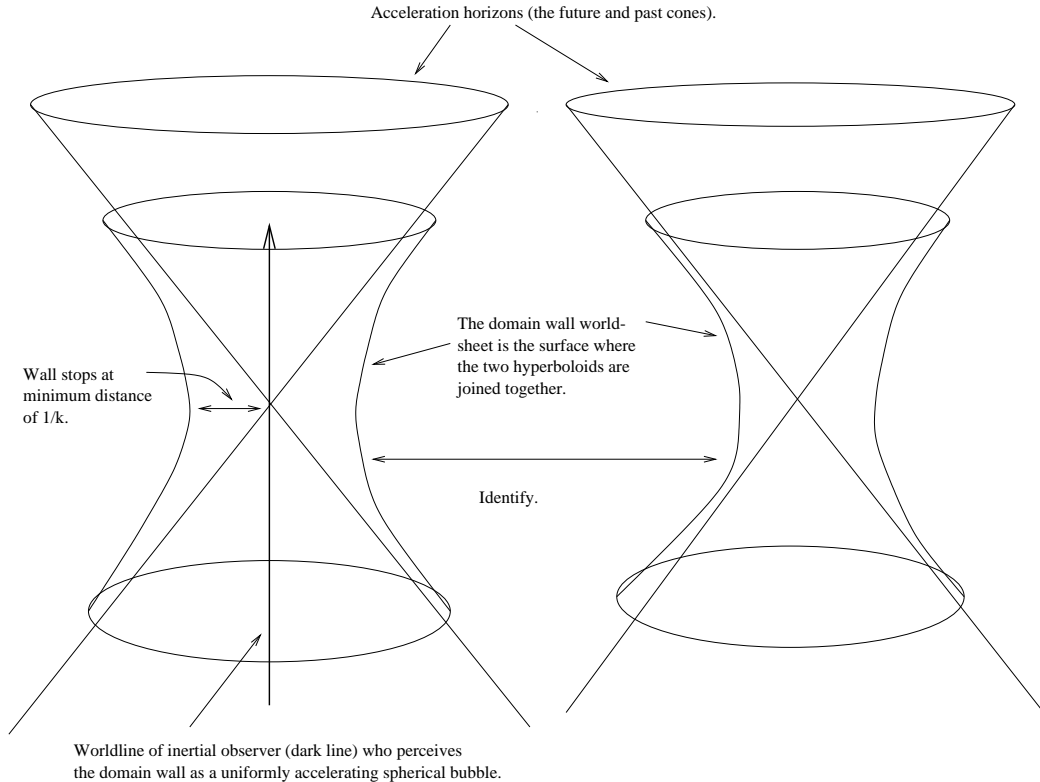


Figure 3

FIG. 3.

Now, the repulsive effect of this vacuum domain wall is very similar to the inflationary effect of a positive cosmological constant seen in de Sitter space. As we noted above, inflation provides an energy source for the pair creation of black hole pairs in the early universe. Similarly, we would expect the repulsive gravitational energy of the VIS domain wall to provide a mechanism for black hole creation and indeed this was shown to be the case in [18]. We will now discuss this process of black hole pair creation in some detail, because it will provide a prototypical example for our general construction.

C. Black Hole Pair Creation on a Domain Wall Background

In [18] the creation rates of charged and uncharged black hole pairs on a VIS background were calculated using the No-Boundary Proposal. Thus, amplitudes were calculated by first finding the Euclidean action of the initial state (consisting of a single domain wall with no black holes present), then finding the Euclidean action of the final state (describing a domain wall with a black hole on each side) and then applying equation (2.4) to obtain the correct rate. The actual black hole creation process which was studied is illustrated in Fig. 4.

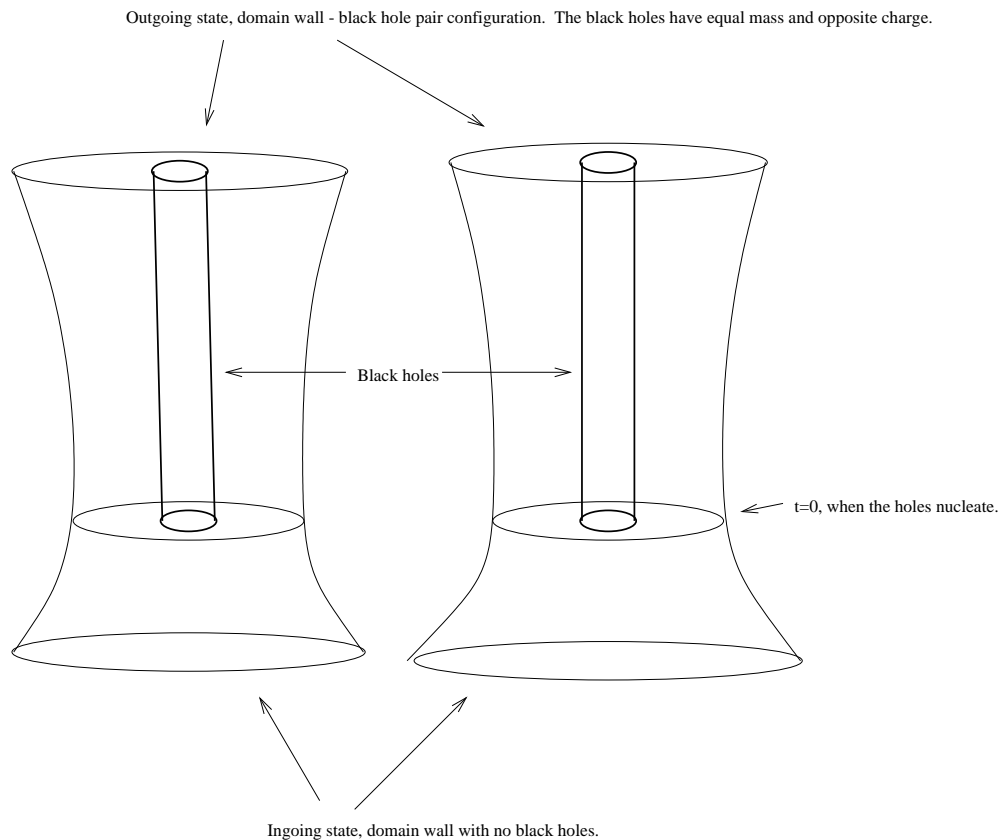


FIG. 4. *A pair of black holes nucleated via the repulsive gravitational energy of the VIS domain wall.*

(Actually, in [18] the only creation process studied was that of black holes which are spher-

ically centered on the domain wall, i.e., each black hole was required to sit exactly in the center of each side of the original spherical domain wall. This is because the motion of a domain wall which is spherically centered on a black hole is known in analytic form. Of course, one could also study the pair creation of black holes which are ‘off center’, but it is likely that numerical methods would be required to simulate the exact wall motion after the black holes were created.)

The instanton, or Euclidean section, of the VIS solution is very similar to the S^4 instanton which mediates the creation from nothing of a de Sitter universe. This instanton allows us to calculate the rate at which the initial state, with no black holes, will be created from nothing. Since the Lorentzian section of the VIS configuration is just two portions of flat Minkowski spacetime glued together, a natural ‘guess’ for the Euclidean section is to take two flat Euclidean four-balls and glue them together along a common (S^3) boundary. When we do this we obtain the so-called ‘lens instanton’, which acquires its name from the fact that it looks rather like a lens with a ridge of curvature running along the hemisphere where the domain wall, D , is located, as illustrated in Fig. 5.

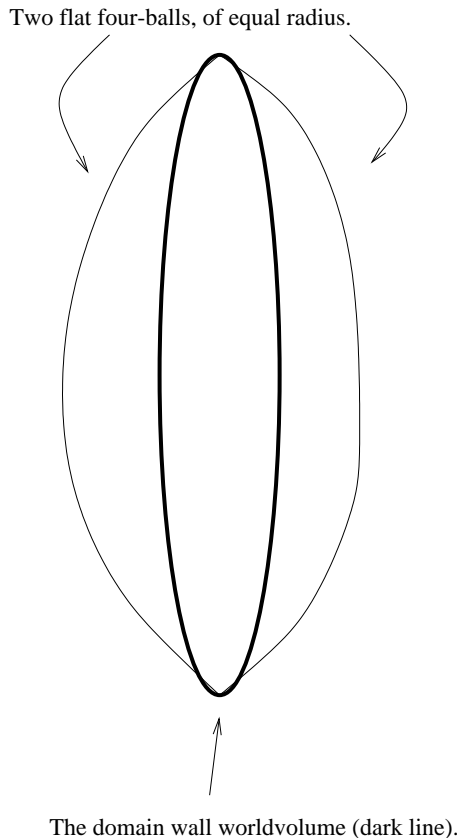


FIG. 5. *The lens instanton is obtained by gluing two flat four-balls together along their respective S^3 boundaries.*

Using equation (3.3) above, one calculates [18] that the total Euclidean action of this instanton is

$$I_D = -\frac{\sigma}{2} \int_D d^3x \sqrt{h} = \frac{-1}{8\pi\sigma^2}, \quad (3.9)$$

where $\int_D d^3x \sqrt{h}$ is the volume of the domain wall worldsheet (the ‘ridge’) on the instanton, and we have used the fact that the radius, r , of each four-ball is given in terms of the energy density as

$$r = \frac{1}{2\pi\sigma}. \quad (3.10)$$

Note that the energy density $\sigma = 1/(2\pi r)$ is manifestly positive here because of the sign of the extrinsic curvature as one moves across the domain wall worldsheet. More explicitly, the extrinsic curvature of a sphere of radius r in flat space is of course $\pm 1/r$, where the sign is determined by whether one is calculating relative to the outward or inward normal to the surface. As one approaches the VIS domain wall from one side, the 3-spheres are locally ‘expanding’ and so the extrinsic curvature is given as $K_{ij}^+ = +(1/r)h_{ij}$. Likewise, as one recedes from the domain wall on the other side the 3-spheres are locally contracting, and so the extrinsic curvature has the sign $K_{ij}^- = -(1/r)h_{ij}$. Using condition (2) of the Israel matching conditions described above we thus see that the energy density satisfies $(1/r)h_{ij} - (-(1/r)h_{ij}) = 4\pi\sigma h_{ij}$, from which Eq. (3.10) follows. The reason we are emphasizing this point is that one can reverse the sign of the energy density simply by considering domain wall worldsheets where the extrinsic curvature has the opposite behavior as one moves through the domain wall. For a domain wall of negative energy density, as one approaches the wall from one side the spheres will be locally *contracting*, and when one leaves the wall from the other side the spheres will start expanding again. Thus, locally the Euclidean section of such a domain wall would look rather like a ‘yo-yo’, with the domain wall worldsheet running along the groove for the string, as illustrated in Fig. 6.

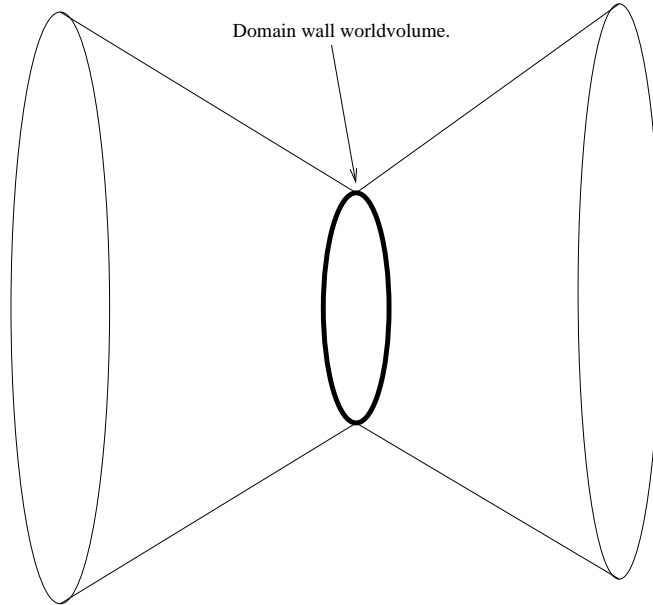


FIG. 6. *Instanton (locally) for a negative energy density domain wall.*

We now need to describe the Euclidean section of the ‘final state’, which consists of a pair of black holes moving relative to the domain wall. As discussed in [18], the motion of black holes relative to a thin domain wall was worked out long ago by Berezin et al. [26] and

Hiscock [27], for the case where the black holes are spherically centered in the middle of each side of the domain wall. The exact equations were presented in [18] and are not important for our analysis here. Here we simply sketch the main physical properties of a black hole - domain wall configuration.

If the created black holes each carry a single $U(1)$ charge, then the only physical parameters in the problem are the masses of the holes (assumed to be equal), the charges of the holes (assumed to be opposite and equal) and the energy density of the wall. Since the domain wall is repulsive and the black holes attract each other, there are basically three cases:

Case 1: The repulsive energy density of the wall overwhelms the attractive force between the holes and the black holes continue to move apart after they have been created.

Case 2: The repulsive energy of the wall exactly counterbalances the attraction between the black holes and the final configuration is in static equilibrium.

Case 3: The attractive force between the holes is greater than the repulsive force of the domain wall and the holes eventually crash together.

While all of these possibilities are in principle allowed, we will focus on Case 2 since in that situation the construction of the instanton is much simpler. It will be clear, however, that everything we say here will also go through for the other cases. As discussed in [18], a solution always exists for the motion of a static domain wall relative to a black hole. If the black hole has mass m and charge $\pm q$, then the (totally umbilic) domain wall hypersurface lies at the constant radius r_s given by

$$r_s = \frac{3}{2}m \left[1 + \sqrt{1 - \frac{8}{9} \frac{q^2}{m^2}} \right].$$

Thus the final state, consisting of two black holes of opposite charge separated by a static spherical domain wall, is obtained by taking two copies of Reissner-Nordström, cutting each copy along a timelike cylinder at $r = r_s$, then gluing the two solid interiors of the cylinders along the domain wall hypersurface. The Euclidean section for this configuration is therefore obtained by taking two ‘cigar instantons’ (for Reissner-Nordström spacetime), snipping each cigar along the hypersurface $r = r_s$ (taking care to keep the ‘tip’ of each cigar, where the black hole horizons are), then gluing the two ends of the cigars together along this surface. The action of this instanton is calculated to be

$$I_{\text{Dbh}} = -2\pi\sigma r^2 \beta_{\text{RN}} \tilde{f}^{1/2} \Big|_{r_s} + q^2 \beta_{\text{RN}} \left(\frac{1}{r_+} - \frac{1}{r_s} \right), \quad (3.11)$$

where β_{RN} is the period of the Reissner-Nordström cigar instanton, r_+ denotes the outer black hole horizon radius, and $\tilde{f} = 1 - 2m/r + q^2/r^2$.

Given this, we can apply the No-Boundary Proposal and Eq. (2.4) to obtain the probability that static black hole pairs (of mass m and charge $\pm q$) will be nucleated by a VIS domain wall:

$$P = \frac{P_{\text{BH}}}{P_{\text{noBH}}} = \exp \left[-\frac{1}{8\pi\sigma^2} + 2\pi\sigma r_s^2 \beta_{\text{RN}} \tilde{f}^{1/2} - q^2 \beta_{\text{RN}} \left(\frac{1}{r_+} - \frac{1}{r_s} \right) \right] \quad (3.12)$$

Similar probabilities are obtained when the created holes are allowed to accelerate relative to the domain wall (the only subtlety in the non-static situation is that there are non-trivial matching conditions which the Euclidean sections of the black hole - domain wall

configurations must satisfy in order to be well-defined instantons). Clearly, the probability is heavily suppressed when the wall energy density σ is small, as would be expected.

Using the No-Boundary Proposal, we have described the calculation of a (generic) probability that stationary black hole pairs will be created in the presence of a VIS domain wall. However, this approach has not told us how to construct an imaginary time path connecting the initial and final states. We will now show how to construct such a path which contains an off-shell wormhole fluctuation of arbitrarily large energy density, but arbitrarily small action.

IV. HOW TO BUILD INTERPOLATING PATHS

When one uses the No-Boundary Proposal to calculate a tunneling amplitude in a cosmological scenario, what one does conceptually is calculate the rate at which one universe will annihilate, and another universe will be created, in its place. Here, we will describe how to ‘patch up’ this picture by performing surgery on the no-boundary instantons to obtain a connected path which connects the ingoing and outgoing universes.

The surgery is actually quite simple, and it is closely related to the operation of taking the connected sum of two manifolds in topology. The only subtlety here is that we want to take the connected sum in such a way that the surface along which the manifolds are joined satisfies the Israel matching conditions, so that we can interpret the matching surface as a virtual domain wall. The resulting manifold will then be an off-shell history which connects the ingoing state to the outgoing state.

In order to have an explicit example where we can implement this construction, let us return to the above scenario where black holes are created on the VIS background. As we saw above, the instanton for the initial state was half of the ‘lens instanton’ (M_L), which was obtained by gluing two flat four-balls together. Likewise, the instanton for the final state was half the ‘baguette’ instanton (M_B), obtained by gluing the ends of two cigar instantons together. We now patch these two instantons together by first removing a (solid) four-dimensional ball, $B^4(\delta)$, of radius δ , from each of the instantons. The boundary components left behind once we remove these small balls are then totally umbilic three-spheres of equal radius; we now join the two instantons together along these three-spheres in such a way that the joining surface satisfies the Israel matching conditions. This construction is illustrated

in Fig. 7.

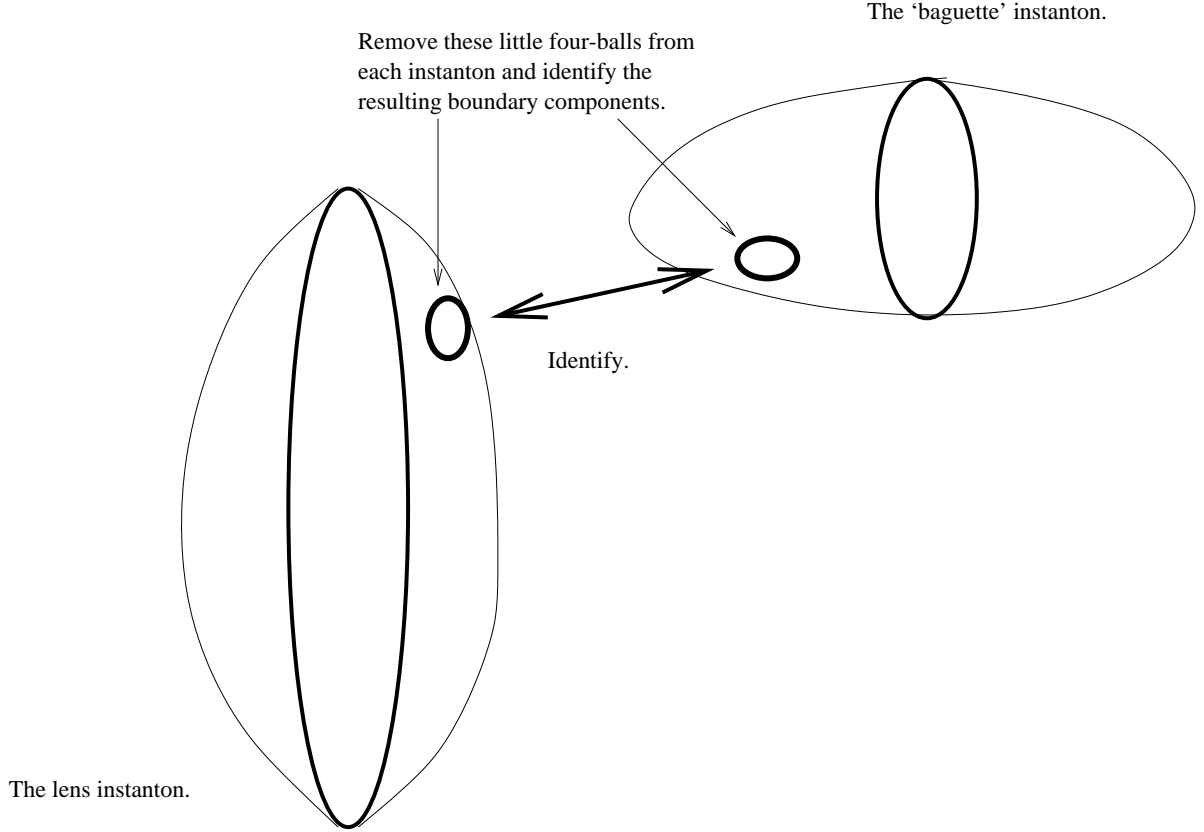


FIG. 7. *Gluing the instantons together along the virtual domain wall, or thin wormhole.*

In this way we obtain an interpolating history which models the decay of of VIS domain wall to a VIS - black hole system. We will show below that the action for this path will be very similar to the action difference of the two instantons used in the No-Boundary Proposal.

However, before turning to this calculation we should point out that the energy density of the virtual wormhole must be *negative*. This follows easily from the analysis presented above: As one approaches the wall hypersurface from one side, neighboring umbilic three-spheres are shrinking, and as one leaves the wall on the other side the three-spheres are expanding. Thus, the three-sphere corresponding to the domain wall worldsheet (which we interpret as a ‘virtual’ domain wall of topology S^2 which appears from nothing, expands briefly, then annihilates) has negative energy density. Indeed, the energy density, which we denote by $\bar{\sigma}$, is set by the scale of the virtual domain wall; it is calculated to be

$$\bar{\sigma} = \frac{-1}{2\pi\delta}. \quad (4.1)$$

Regions of negative energy are perfectly allowed for off-shell paths such as the ones we are constructing here. There are two reasons why one should not attempt a similar prescription using real domain walls (on-shell paths): First, it would require assumptions about the matter fields – they would have to allow domain walls. This would make the analysis less general. Second, there is no sensible classical field theory which can give rise to domain walls of negative energy density, because this would destroy vacuum stability.

Of course, this may seem confusing given that wormholes supported with large amounts of negative energy density are considered every day by the ‘wormhole engineers’ [28], who need the negative energy density in order to make the wormholes traversable. However, these engineers use quantum mechanical processes (such as the Casimir effect) to construct regions with large negative energy density. We will not consider such issues here.

Given this off-shell resonance, we now want to calculate the action and get a tunneling amplitude. This calculation is fairly straightforward once we notice several elementary points.

First of all, there are no volume contributions to the action in this particular scenario, and so we don’t have to worry about the fact that we have removed four-balls from the original instantons (as expected, the only contribution there will come from the virtual wormhole itself). Actually, as we shall show in a moment that one never has to worry about the volume contributions (even when there is a cosmological constant, for example) since the volume terms from the removed balls will always cancel each other.

Second, the virtual domain wall hypersurface is crucial because it ensures that the initial and final instantons will have *opposite* relative orientations. This means that the two actions will appear with the correct relative signs.

In order to understand this orientation change, it is useful to consider what happens when you try to extend a (C^0) tetrad field through the surface of the domain wall. To be concrete, assume that the tetrad fields on each of the little balls removed from the the instantons are comprised of the vectors naturally associated with polar normal coordinates on each ball. That is to say, three vectors of a tetrad are taken to be (angular) coordinates tangent to the boundary of a ball, and the fourth vector is a (radial) vector normal to the boundary of a ball. Now, if we want the tetrads on each ball to ‘match’ at the surface of the domain wall, it follows that we *must* take the normal component of the tetrad to be inward pointing on one ball and outward pointing on the other. Thus, as we move from one instanton to the next we reflect about one leg of the tetrad, i.e., we reverse the orientation.

Indeed, this is precisely why we don’t have to worry about the removed volume contributions. If the removed four-balls have equal, but opposite action, then the final difference (after they have been removed) is equal to the original No-Boundary Proposal difference. Of course, this cancellation actually only works when the four-balls have *exactly* equal action contributions. One might imagine a situation where, for example, the cosmological constant ‘jumped’ to a different value during the decay process (as you moved from the initial instanton to the final instanton across the virtual domain wall worldsheet). In such a scenario, the actions of the two removed four-balls would not be equal and so there would be an extra correction term to the no-boundary result. However, we will not consider such complications in this paper.

Given these comments, it is now clear that the final action, I_T , for our interpolating path is just the original no-boundary difference plus a small correction term coming from the virtual domain wall:

$$I_T = \frac{1}{2}I_{\text{Dbh}} - \frac{1}{2}I_D + \frac{1}{8\pi\bar{\sigma}^2} \quad (4.2)$$

where I_{Dbh} and I_D are given by (3.11) and (3.9) respectively and the correction term involving $\bar{\sigma}$ appears with a plus sign because the virtual domain wall has a negative energy density.

Taking the point of view of the bounce approach (see Sec. II A) one should complete the newly connected half-instantons with their mirror image to obtain a bounce, in which one starts with the initial spacelike section, goes through a final one and back to the initial type at the other end of the Euclidean geometry. This requires another surgery in Fig. 7, with a second virtual domain wall allowing the transition from the baguette back to the lens. The total action will obviously be twice that in Eq. (4.2). Using Eq. (4.1), it may be written in the form

$$I(\delta) = I_0 + \frac{1}{2}\rho\delta^2, \quad (4.3)$$

where

$$I_0 = I_{\text{Dbh}} - I_{\text{D}}, \quad \rho = 2\pi. \quad (4.4)$$

The transition rate will be given by

$$\Gamma = \int_{-\infty}^{\infty} d\delta e^{-I} \quad (4.5)$$

$$= e^{-I_0} \int_{-\infty}^{\infty} d\delta e^{-\frac{1}{2}\rho\delta^2} \quad (4.6)$$

$$= e^{-I_0}. \quad (4.7)$$

We demand that the disconnected geometry be excluded from this path, and we assume that the connecting virtual domain wall must have a diameter of at least the Planck length. This will restrict the range of integration in Eq. (4.6) to the regions of more than one standard variation, and will therefore reduce the value of the prefactor from 1 to about 1/3. Given the exponential suppression of black hole pair creation, this change is negligible. Therefore, the requirement for connected interpolating geometries will not alter the no-boundary approach to cosmological pair creation significantly: The exponent will be unchanged, and the prefactor, which is usually neglected anyway, will still be of the same order of magnitude. Our approximation breaks down only for Planck-scale background geometries, when the semi-classical approach fails in any case.

Of course, one can also apply this construction to other gravitational tunneling phenomena, such as black hole pair creation on a cosmological background. There again, as long as the cosmological constant is conserved in the decay process, the final answer will be the original no-boundary result with a reduced prefactor.

One might object to the use of off-shell paths. However, they are not only an essential part of the formal path integral formalism, but they give a significant contribution to the saddlepoint approximation – after all, the actual saddlepoint solution forms a set of measure zero in the saddle point contour. Crucially, we demonstrated that the interpolating off-shell paths we considered are in fact arbitrarily small perturbations of the saddlepoint solution. Thus, if the saddlepoint is excluded from the integral, these geometries will still dominate. Therefore we stress that on the contrary, our proposal offers a consistent way of including the effects of spacetime foam in semi-classical calculations.

We should also point out here that our construction is in many respects similar to the earlier work of Farhi, Guth and Guven [29], who were studying the rate at which a new universe could be created in the laboratory. In their approach, one constructs an interpolating

instanton by gluing the two instantons together to obtain a ‘two-sheeted’ pseudomanifold. The basic idea there is that the ingoing state is on one sheet, and the outgoing state is on the other sheet. This approach was recently employed by Kolitch and Eardley, who studied the decay of vacuum domain walls. Interestingly, they found that the rate calculated using the Farhi, Guth, Guven (FGG) technique is identical to the rate calculated using the No-Boundary Proposal. Thus, it would seem that attempts to construct some interpolating geometry (in situations where no connected, on-shell path exists) will always lead back to the no-boundary ansatz.

The current debate about the correct boundary conditions on the wave function of the universe [30] centers on the question of whether the Hartle-Hawking No-Boundary Proposal, or the Tunneling Proposal favored by Linde [32], Vilenkin [33] and others, should be used to describe the creation of a universe from *nothing*. We emphasize here that the use of the No-Boundary Proposal for the tunneling processes on an *existing* background is not called into doubt by either Hawking or Linde. Here we have aimed to elucidate the reasons for the success of the No-Boundary Proposal for such processes.

Finally, in light of recent work [31] concerning the possible role of singular instantons in describing tunneling phenomena, we would like to emphasize that the interpolating paths which we have constructed here are in no way singular. Rather, these trajectories simply contain off-shell fluctuations which may be regarded as distributional sources of negative energy density.

V. SUMMARY

In this paper we have aimed to justify the use of compact instantons for the semiclassical description of tunneling processes on cosmological backgrounds. We found connected off-shell interpolating geometries which can be viewed as small perturbations of the disconnected on-shell instantons. Therefore they can dominate the path integral. Since the disjoint solutions are linked by a virtual domain wall in our approach, the total action of the interpolating geometry will be the *difference* between the instanton actions, and we recover the result obtained from the No-Boundary approach.

Over the years, black hole pair creation has been investigated on a variety of backgrounds. Usually, one finds a Euclidean bounce solution which includes spacelike sections with and without black holes. The difference between the bounce action and the action of the Euclidean background solution is calculated. The pair creation rate is obtained as the exponential of minus this action difference, as in Eq. (2.1).

This method fails for the cosmological and domain wall backgrounds we have considered, because there is no single Euclidean solution that will interpolate between the initial and final spacelike sections. Instead, one can construct two nucleation geometries (half-bounces, which are continued in a Lorentzian direction rather than back to the initial spacelike section), which both describe the nucleation of a universe from nothing: one with black holes, the other without. Therefore, the pair creation problem can be attacked within the framework of quantum cosmology. This leads to a prescription for the pair creation rate of cosmological black holes.

This ‘quantum cosmological’ approach rests on the assumption that each Hubble volume in an inflationary universe can be regarded as having been nucleated independently. In

quantum cosmology, one constructs a wave function of the universe. The square of its amplitude gives a probability measure. By taking the ratio of the probability measures assigned to the two instanton types, one can calculate the relative probability for a Hubble volume to nucleate with a black hole pair, compared to an empty Hubble volume. Unless the cosmological constant is of Planck order, this number is small, and can be interpreted as a pair creation rate in the natural length and time scale, the Hubble scale.

One then uses the Hartle-Hawking No-Boundary Proposal to determine the wave function semi-classically. According to this proposal, the probability measure will be given by the exponential of minus twice the real part of the instanton action, which is equivalent to the full bounce action of the usual pair creation treatment. The ratio of these two exponentials is, of course, equivalent to a single exponential of the action difference. Thus, one recovers the usual prescription for the pair creation rate, as far as is possible given the fundamental differences between the cosmological and the non-cosmological situations. This is an important test for consistency, especially since the instanton actions reflect the geometric entropy of the nucleated spacetimes. Schwarzschild-de Sitter space has a lower geometric entropy than de Sitter space. Thus the instanton actions reflect the physical necessity that transitions in the direction of lower entropy are suppressed.

While this application of the No-Boundary Proposal to decay processes in quantum cosmology would seem to be intuitively justified by these arguments, it seemed to rely on two disconnected instantons, instead of describing the transition through a single Euclidean geometry connecting the initial state to the final state.

In this paper, we showed that the exact saddlepoint solution is a special disconnected geometry in a generic class of connected ones, in which the instantons are patched together using virtual domain walls. These off-shell geometries can be arbitrarily close to the saddlepoint and contribute to its domination of the path integral. The exclusion of disconnected geometries therefore has no fundamental effect on the formalism. The exponential suppression of the pair creation process is left unchanged, and we estimated that the prefactor would be diminished by a factor of a third.

One could also use our method to study other decay processes which are expected to occur in the early universe. A natural candidate process would be the decay of vacuum domain walls by quantum tunneling recently discussed by Kolitch and Eardley ([23], [34]). This process is important because it provides another decay mode capable of eliminating the unwanted gravitational effects of domain walls in the early universe. It is also possible to generalize the Kolitch-Eardley analysis to supergravity domain walls (such as the D8-branes of the IIA theory) which arise in the low-energy limit of string theory [35]. Research on these and related problems is currently underway.

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